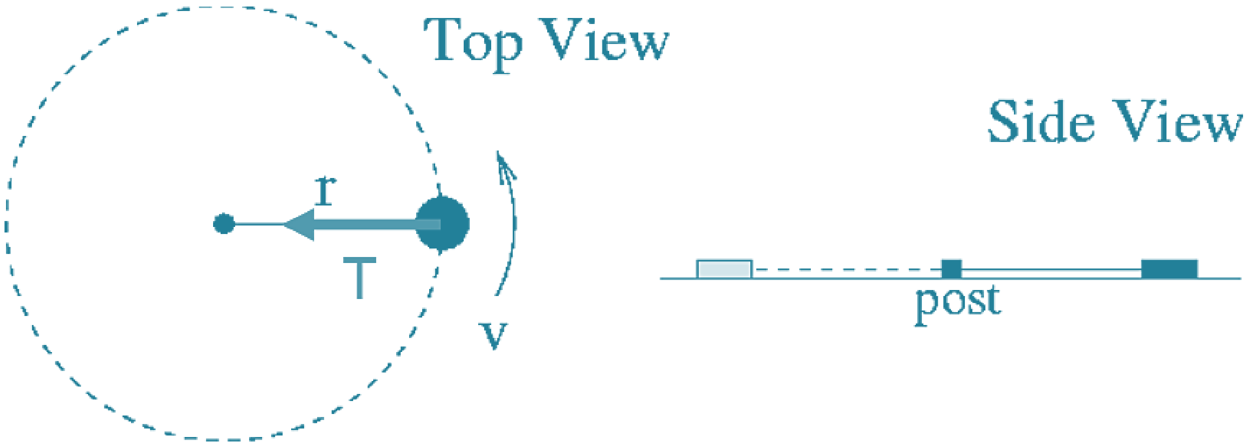
12.2 (a) Centripetal acceleration and force (examples)
Example 12.2.1

Motion in a Horizontal Circle:

i. Suppose that a mass is tied to the end of a string and is being whirled in a circle along the top of a frictionless table as shown in the diagram.

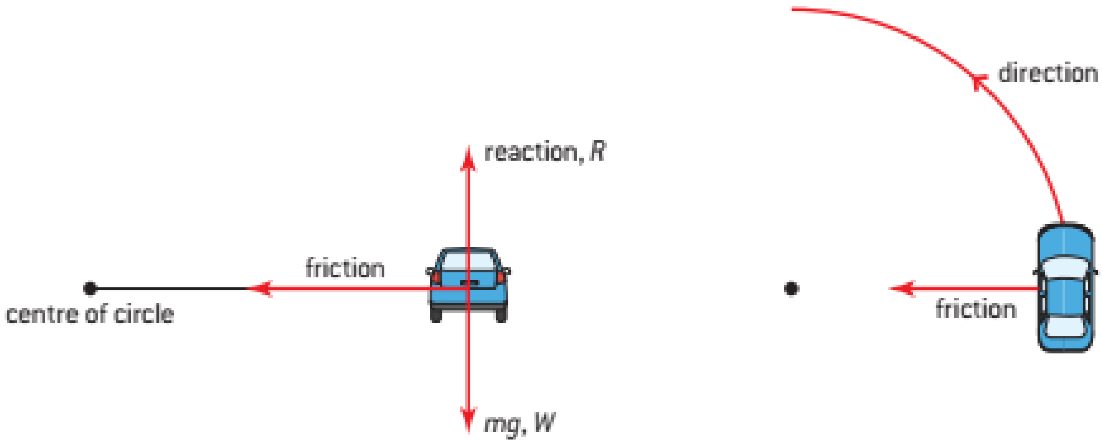


A free body diagram of the forces on the mass would show that the tension is the centripetal force:

$$F_c = T = \frac{mv^2}{r}$$

Where T is the tension in the string.

. ii. Suppose that a car is moving in a circle along a circular track as shown in the diagram.



The free body diagram shows that the resultant force acting on the car towards the centre is the frictional force fs between the road and tyres of the car.

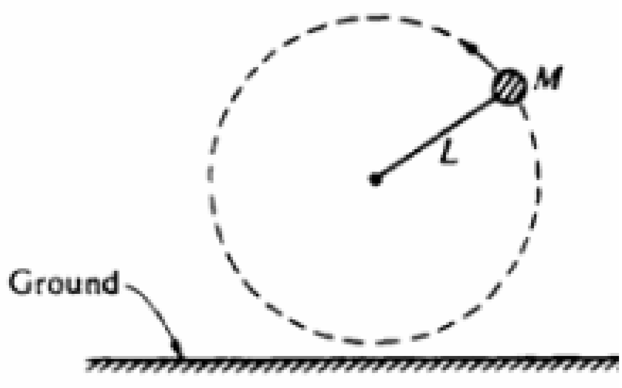
Therefore $F_c = f_s = \frac{mv^2}{r}$

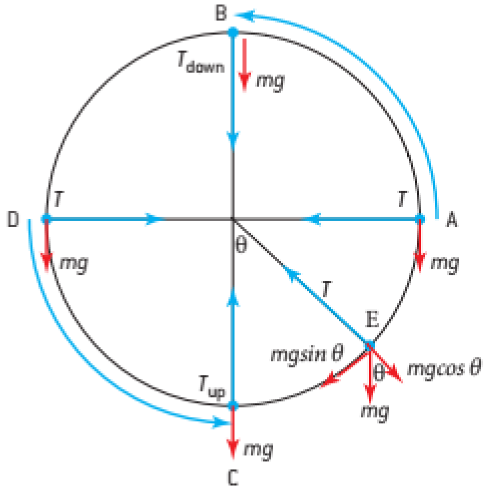
Example 12.2.2

Motion in a Vertical Circle: Unlike horizontal circular motion, in vertical circular motion the speed, as well as the direction of the object, is constantly changing. Gravity is constantly either speeding up the object as it falls, or slowing the object down as it rises.

i. Mass attached to string

Suppose an object of mass m is being whirled on the end of a string in a vertical circle. Let's look at the free body diagrams of the forces acting on the block at the top of the circle and at the bottom of the circle.





Top of the circle

At point B the forces T (tension) and mg (weight) are directed downwards towards the center. Since the mass is in circular motion, therefore, the net force must act towards the center of the circle.

the centripetal force is the sum of T_{down} and mg.

$$F_c = T_{down} + mg$$

$$T_{down} + mg = \frac{mv^2}{r}$$

$$T_{down} = \frac{mv^2}{r} - mg$$

To calculate the minimum or critical velocity(v_{min}) needed for the block to just be able to pass through the top of the circle without the rope sagging, let the tension in the rope approaches zero.

$$0 = \frac{mv^2}{r} - mg$$

$$\frac{mv^2}{r} = mg$$

$$v_{min} = \sqrt{rg}$$

Bottom of the circle

At the bottom of the circle at point C. The net force acting towards the center, F_c, is the difference between T_{up} and mg as they point in opposite directions.

centripetal force = T_{up}- mg

$$F_c = T_{up} - mg$$

$$T_{up} - mg = \frac{mv^2}{r}$$

$$T_{up} = \frac{mv^2}{r} + mg$$

This formula will be used frequently to calculate the tension in the string in a simple pendulum as the pendulum bob swung through its lowest position - the equilibrium position, the point of greatest KE.

Note: the tension in the string is GREATEST as the block passes through the bottom of the circle and LEAST while it passes through the top of the circle.

Intermediate Point at angle θ with the vertical

If you were asked to calculate the tension at any **intermediate point**, then the net force i.e the centripetal force would be equal to T - mg cos θ.

$$F_c = T - mgcos\theta$$

$$T - mgcos\theta = \frac{mv^2}{r}$$

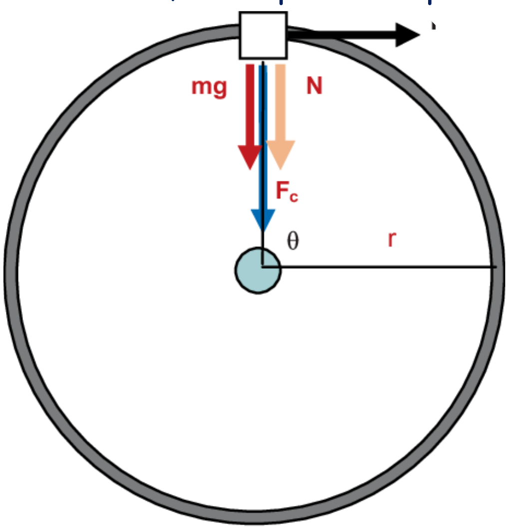
$$T = \frac{mv^2}{r} + mgcos\theta$$

Notice that components of the weight are taken, not components of the tension. It is a component of the weight vector that acts along the tangent and produces acceleration.

In order to solve for tension, you would have to use conservation of energy techniques to first solve for the velocity at the requested intermediate position.

ii. **Roller Coaster**

Now, consider an example of a person riding a roller coaster through a circular section of the track, a "loop-the-loop."



Let's look at the formulas needed to calculate the **normal force**, N, exerted on a object traveling on the inside surface of a vertical circle as it passes through the bottom and through the top of the ride.

At the top

The centripetal force is the sum of N (normal reaction) and mg (weight)

$$F_c = N + mg$$
$$N + mg = \frac{mv^2}{r}$$
$$N = \frac{mv^2}{r} - mg$$

If we let the value of normal approach zero in the formula the same value for the critical velocity that we got when solving for the tension in the string in our previous discussion, $v_{min} = \sqrt{rg}$

In roller coasters, this critical velocity is a safety threshold.

At the bottom

The centripetal force is the resultant of N (normal reaction) and mg (weight)

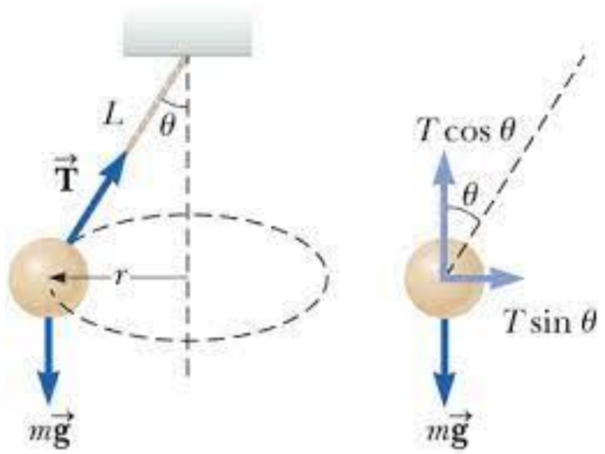
$$F_c = N - mg$$
$$N - mg = \frac{mv^2}{r}$$
$$N = \frac{mv^2}{r} + mg$$

Note: that the normal reaction, N, appears to play the same role as the tension, T, in our equations for vertical circular motion.

Example 12.2.3

Motion of conical pendulum:

An object of mass 'm' attached to the end of string making angle θ with the verticle and tracing a horizontal circle is a conical pendulum. This is a horizontal circle in which gravity is always perpendicular to the object's path.



Free body diagram of the mass on the end of the pendulum shows the following forces. Tcos θ balanced by the object's weight mg and Tsinθ is supplying the centripetal force necessary to keep the block moving in its circular path:

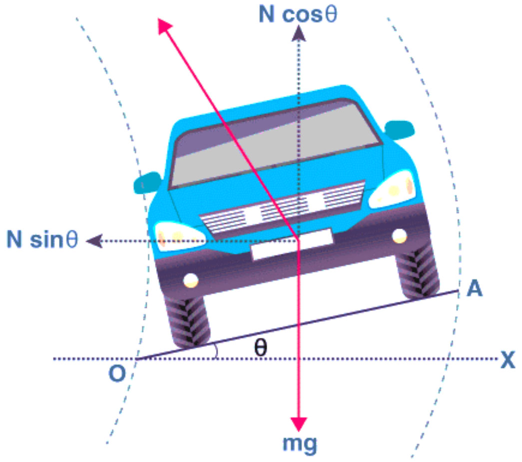
$$F_c = T\sin\theta$$
$$T\sin\theta = \frac{mv^2}{r}$$

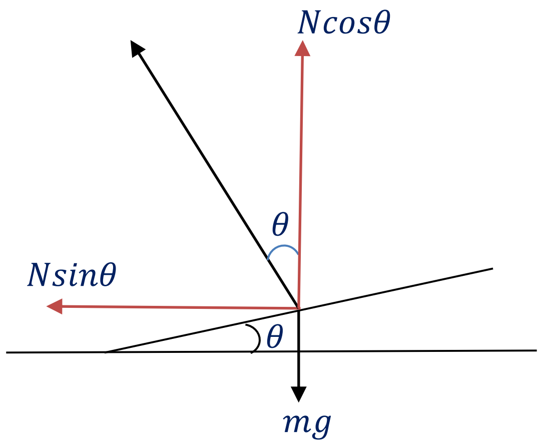
and $T\cos\theta = mg$

Example 12.2.4

Banking

Tracks for motor or cycle racing, and even ordinary roads for cars are sometimes **banked**. The curve of the banked road surface is inclined at an angle so that the normal reaction force contributes to the centripetal force that is needed for the vehicle to go round the track at a particular speed. Tyres do not need to provide so much friction on a banked track compared to a horizontal road,this reduces the risk of skidding and increases safety.





Free body diagram of the forces acting on the car shows weight (mg) and a normal reaction force (N). As the car is not sliding down the bank of the incline, but is instead traveling across the incline, components of the normal are examined.

N sin θ is the unbalanced central force;,

This component of the normal is supplying the centripetal force necessary to keep the car moving through the banked curve.

$$F_c = Nsin\theta$$

$$N\ sin\ \theta = \frac{mv^2}{r}$$

and

$$Ncos\theta = mg$$

the above two equations yield

$$tan\theta = \frac{v^2}{rg}$$

Solving for v produces

$$v_{critical} = \sqrt{rg\ tan\theta}$$

At this critical speed, there is no need for any friction between the car and the road's surface. If the speed of the car were to exceed $v_{critical}$ then the car would drift up the incline. If the speed of the car is less than $v_{critical}$ then the car would slip down the incline.